

# Friday 20 January 2012 – Afternoon

# **A2 GCE MATHEMATICS (MEI)**

4768 Statistics 3

**QUESTION PAPER** 

Candidates answer on the Printed Answer Book.

## **OCR** supplied materials:

- Printed Answer Book 4768
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

• Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

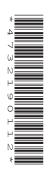
## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 (a) Define simple random sampling. Describe briefly one difficulty associated with simple random sampling. [4]
  - **(b)** Freeze-drying is an economically important process used in the production of coffee. It improves the retention of the volatile aroma compounds. In order to maintain the quality of the coffee, technologists need to monitor the drying rate, measured in suitable units, at regular intervals. It is known that, for best results, the mean drying rate should be 70.3 units and anything substantially less than this would be detrimental to the coffee. Recently, a random sample of 12 observations of the drying rate was as follows.

66.0 66.1 59.8 64.0 70.9 71.4 66.9 76.2 65.2 67.9 69.2 68.5

- (i) Carry out a test to investigate at the 5% level of significance whether the mean drying rate appears to be less than 70.3. State the distributional assumption that is required for this test. [10]
- (ii) Find a 95% confidence interval for the true mean drying rate. [4]
- 2 In a particular chain of supermarkets, one brand of pasta shapes is sold in small packets and large packets. Small packets have a mean weight of 505 g and a standard deviation of 11 g. Large packets have a mean weight of 1005 g and a standard deviation of 17 g. It is assumed that the weights of packets are Normally distributed and are independent of each other.
  - (i) Find the probability that a randomly chosen large packet weighs between 995 g and 1020 g. [3]
  - (ii) Find the probability that the weights of two randomly chosen small packets differ by less than 25 g. [3]
  - (iii) Find the probability that the total weight of two randomly chosen small packets exceeds the weight of a randomly chosen large packet. [4]
  - (iv) Find the probability that the weight of one randomly chosen small packet exceeds half the weight of a randomly chosen large packet by at least 5 g. [4]
  - (v) A different brand of pasta shapes is sold in packets of which the weights are assumed to be Normally distributed with standard deviation 14 g. A random sample of 20 packets of this pasta is found to have a mean weight of 246 g. Find a 95% confidence interval for the population mean weight of these packets.

    [4]

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3 (a) A medical researcher is looking into the delay, in years, between first and second myocardial infarctions (heart attacks). The following table shows the results for a random sample of 225 patients.

Delay (years)	0 –	1 –	2 –	3 –	4 – 10
Number of patients	160	40	13	9	3

The mean of this sample is used to construct a model which gives the following expected frequencies.

Delay (years)	0 –	1 –	2 –	3 –	4 – 10
Number of patients	142.23	52.32	19.25	7.08	4.12

Carry out a test, using a 2.5% level of significance, of the goodness of fit of the model to the data. [9]

**(b)** A further piece of research compares the incidence of myocardial infarction in men aged 55 to 70 with that in women aged 55 to 70. Incidence is measured by the number of infarctions per 10000 of the population. For a random sample of 8 health authorities across the UK, the following results for the year 2010 were obtained.

Health authority	A	В	С	D	Е	F	G	Н
Incidence in men	47	56	15	51	45	54	50	32
Incidence in women	36	30	30	47	54	55	27	27

A Wilcoxon paired sample test, using the hypotheses  $H_0$ : m = 0 and  $H_1$ :  $m \neq 0$  where m is the population median difference, is to be carried out to investigate whether there is any difference between men and women on the whole.

(i) Explain why a paired test is being used in this context.

[1]

(ii) Carry out the test using a 10% level of significance.

[8]

[Question 4 is printed overleaf.]

- 4 At the school summer fair, one of the games involves throwing darts at a circular dartboard of radius *a* lying on the ground some distance away. Only darts that land on the board are counted. The distance from the centre of the board to the point where a dart lands is modelled by the random variable *R*. It is assumed that the probability that a dart lands inside a circle of radius *r* is proportional to the area of the circle.
  - (i) By considering P(R < r) show that F(r), the cumulative distribution function of R, is given by

$$F(r) = \begin{cases} 0 & r < 0, \\ \frac{r^2}{a^2} & 0 \le r \le a, \\ 1 & r > a. \end{cases}$$
 [3]

- (ii) Find f(r), the probability density function of R. [2]
- (iii) Find E(R) and show that  $Var(R) = \frac{a^2}{18}$ . [7]

The radius a of the dartboard is 22.5 cm.

- (iv) Let  $\overline{R}$  denote the mean distance from the centre of the board of a random sample of 100 darts. Write down an approximation to the distribution of  $\overline{R}$ .
- (v) A random sample of 100 darts is found to give a mean distance of 13.87 cm. Does this cast any doubt on the modelling?



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# **A2 GCE MATHEMATICS (MEI)**

4768 Statistics 3

# PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

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- MEI Examination Formulae and Tables (MF2)

## Other materials required:

Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



Candidate forename				Candidate surname			
Centre number				Candidate nu	ımber		

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1 (a)	
1 (b)(i)	
	(answer space continued on next page)

1 (b)(i)	(continued)
,	
,	
,	
,	

1 (b)(ii)	

2 (	
2 (	
`	

2 (iii)	
2 (iv)	
	(answareness sentinued on next ness)
	(answer space continued on next page)

2 (iv)	(continued)
2 (v)	

3 (a)	

3 (b)(i)	
3 (b)(ii)	
3 (b)(ii)	

4 (i)	
4 (ii)	

4 (iii)	

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**GCE** 

# **Mathematics (MEI)**

**Advanced GCE** 

Unit 4768: Statistics 3

# Mark Scheme for January 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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# **Annotations and abbreviations**

Annotation in scoris	Meaning			
√and <b>x</b>				
BOD	Benefit of doubt			
FT	Follow through			
ISW	Ignore subsequent working			
M0, M1	Method mark awarded 0, 1			
A0, A1	Accuracy mark awarded 0, 1			
B0, B1	Independent mark awarded 0, 1			
SC	Special case			
۸	Omission sign			
MR	Misread			
Highlighting				
Other abbreviations	Meaning			
in mark scheme				
E1	Mark for explaining			
U1	Mark for correct units			
G1	Mark for a correct feature on a graph			
M1 dep*	Method mark dependent on a previous mark, indicated by *			
cao	Correct answer only			
oe	Or equivalent			
rot	Rounded or truncated			
soi	Seen or implied			
WWW	Without wrong working			

# Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

### М

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

# В

Mark for a correct result or statement independent of Method marks.

### Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.
  - Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		n	Answer	Marks	Guidance
1 (a			Simple random sampling is when every <u>sample</u> of the required size  has an equal chance of being chosen. eg One needs access to the entire population in order to establish the sampling frame.	E1 E1 E2	SC Allow E1 for "Every member of the population has an equal chance of being chosen".  E2, 1, 0. Reward any sensible point. SC1 "Sample may not be representative of the population" oe.
1 (b	b)	(i)	H <sub>0</sub> : $\mu = 70.3$ H <sub>1</sub> : $\mu < 70.3$ Where $\mu$ is the (population) mean drying rate. $\bar{x} = 67.675$ , $s_{n-1} = 4.129$ ( $s_{n-1}^{2} = 17.049$ ) Test statistic is $\frac{67.675 - 70.3}{\left(\frac{4.129}{\sqrt{12}}\right)}$	B1 B1 B1 M1	Both. Hypotheses in words only must include "population". Do NOT allow " $\overline{X} =$ " or similar unless $\overline{X}$ is clearly and explicitly stated to be a <u>population</u> mean. For adequate verbal definition. Allow absence of "population" if correct notation $\mu$ is used. Do not allow $s_n = 3.953$ ( $s_n^2 = 15.629$ ). Allow c's $\overline{x}$ and/or $s_{n-1}$ . Allow alternative: $70.3 + (\text{c's} - 1.796) \times \frac{4.129}{\sqrt{12}}$ (= 68.15(9)) for subsequent comparison with $\overline{x}$ . (Or $\overline{x} - (\text{c's} - 1.796) \times \frac{4.129}{\sqrt{12}}$ (= 69.81(6)) for comparison with 70.3)
			$= -2.202(2)$ Refer to $t_{11}$ . Lower 5% point is $-1.796$ . $-2.202 < -1.796, \therefore \text{ Result is significant.}$ Seems mean drying rate has reduced. Underlying population is Normal.	A1  M1 A1  A1  A1  [10]	cao but ft from here in any case if wrong. Use of $70.3 - \overline{x}$ scores M1A0, but ft.  No ft from here if wrong. Allow any $t_{11}$ value from tables. c.a.o. No ft from here if wrong.  Must be $-1.796$ unless it is clear that absolute values are being used. $P(t < -2.202) = 0.0249$ . ft only c's test statistic. "Non-assertive" conclusion in context to include "on average" oe.

(	Questic	on	Answer	Marks	Guidance
1	(b)	(ii)	CI is given by 67.675 ±	M1	ZERO if not same distribution as test. Same wrong distribution scores max M1B0M1A0. Recovery to $t_{11}$ is OK. ft c's $\overline{x} \pm$ .
			$ \begin{array}{r} 2.201 \\ \times \frac{4.129}{\sqrt{12}} \end{array} $	B1 M1	ft c's $S_{n-1}$ .
			$= 67.675 \pm 2.623 = (65.05(2), 70.29(8)$	A1 [4]	cao Must be expressed as an interval. ( <i>t</i> = 1.796 gives (65.534, 69.816) (M1B0M1A0)).
2			$S \sim N(505, 11^2)$ $L \sim N(1005, 17^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only.
	(i)		$P(995 < L < 1020)$ $= P\left(\frac{995 - 1005}{17} < Z < \frac{1020 - 1005}{17}\right)$ $= P(-0.5882 < Z < 0.8824)$ $= 0.8113 - (1 - 0.7218)$	M1 A1	For standardising. Award once, here or elsewhere.
			= 0.5331	A1 [3]	cao
2	(ii)		$S_1 - S_2 \sim N(0, 11^2 + 11^2 = 242)$ $P(-25 < S_1 - S_2 < 25)$ $= P\left(\frac{-25 - 0}{\sqrt{242}} < Z < \frac{25 - 0}{\sqrt{242}}\right)$ $= P(-1.607 < Z < 1.607)$ $= 2 \times (0.9459 - 0.5)$ $= 0.8918$	B1 M1	Mean and variance. Accept sd = $\sqrt{242}$ = 15.55 Formulate the problem.
			- 0.8918	A1 [3]	cao

(	Question	Answer	Marks	Guidance
2	(iii)	Want $P(S_1 + S_2 > L)$ i.e. $P(S_1 + S_2 - L > 0)$ $S_1 + S_2 - L \sim N(505 + 505 - 1005 = 5,$ $11^2 + 11^2 + 17^2 = 531)$ $P(\text{this} > 0) = P(Z > \frac{0-5}{\sqrt{531}} = -0.2170)$	M1 B1 B1	Allow $L - (S_1 + S_2)$ provided subsequent work is consistent. Mean Variance. Accept sd = $\sqrt{531}$ = 23.04
		=0.5859	A1 [4]	cao
2	(iv)	Want $P(S > \frac{1}{2}L + 5)$ i.e. $P(S - \frac{1}{2}L > 5)$ $S - \frac{1}{2}L \sim N(505 - 1005/2 = 2.5,$ $11^2 + 17^2/2^2 = 193.25)$ $P(\text{this} > 5) = P(Z > \frac{5 - 2.5}{\sqrt{193.25}} = 0.1798)$	M1 B1 B1	Allow $\frac{1}{2}L - S$ provided subsequent work is consistent. Mean. Variance. Accept sd = $\sqrt{193.25} = 13.90$
		= 1 - 0.5714 = 0.4286	A1 [4]	cao
2	(v)	CI is given by $246 \pm 1.96$ $\times \frac{14}{\sqrt{20}}$ $= 246 \pm 6.1358 = (239.8(6), 252.1(3))$	M1 B1 M1 A1 [4]	Must be 1.96. Anything else can get M1B0M1A0 max.  cao Must be expressed as an interval.
3	(a)	H <sub>0</sub> : The model for the delay fits the data.  H <sub>1</sub> : The model for the delay does not fit the data.  Obs'd frequency 160 40 13  Exp'd frequency 142.23 52.32 19.2  Merge last 2 cells: Obs 12 Exp 11.2 $X^2 = 2.2202 + 2.9010 + 2.0292 + 0.0571$ $= 7.207(5)$ Refer to $\chi^2_2$ .  Upper 2.5% point is 7.378.	B1 B1	Do not allow hypotheses of the form "Data fit model" o.e.  Calculation of $X^2$ . cao. If not merged, $X^2 = 7.975(5)$ No ft if wrong. Allow correct dof (= cells – 2) from wrongly grouped table and ft. Allow any value from tables for c's dof. c.a.o. Upper 2.5% point for c's dof. No ft from here if wrong. $P(X^2 > 7.2075) = 0.0272$ .

(	Question		Answer	Marks	Guidance
			7.207 < 7.378 ∴ Not Significant. Suggests it is reasonable to suppose the model fits the data.	A1 A1 [9]	ft only c's test statistic. ft only c's test statistic. "Non-assertive" conclusion in words (+ context). Do not allow "Data fit model" o.e.
3	<b>(b)</b>	(i)	A paired test is used in this context in order to eliminate differences between health authorities.	E1 [1]	oe
3	(b)	(ii)		23   5 7   3   M1   M1   A1   B1   M1   A1   A1   A1   A1	For differences. ZERO in this section if differences not used. For ranks. ft from here if ranks wrong. (or $W_+ = 2 + 3 + 5 + 7 + 8 = 25$ )  No ft from here if wrong.  ie a 2-tail test. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic.  "Non-assertive" conclusion in context to include "on the whole" oe.
4	(i)		$P(R < r) = k\pi r^{2}$ $P(R < a) = 1  \therefore k = \frac{1}{\pi a^{2}}$ $\therefore P(R < r) = \frac{\pi r^{2}}{\pi a^{2}} = \frac{r^{2}}{a^{2}}.$ Thus $F(r) = \frac{r^{2}}{a^{2}}  \text{(for } 0 \le r \le a\text{)}.$	M1 M1 A1	Formulate probability proportional to area in terms of " $k$ ". Find $k$ .  IF M0M0, allow SC B1 for $P(R < r) = \frac{\pi r^2}{\pi a^2} =$ www Convincingly shown; ANSWER GIVEN. Condone omission of $r < 0$ and/or $r > a$ .
4	(ii)		For $0 \le r \le a$ , $f(r) = \frac{d}{dr} F(r) = \frac{2r}{a^2}$ .	M1 A1 [2]	Condone omission of $r < 0$ and/or $r > a$ .

	Questio	n Answer	Marks	Guidance
4	(iii)	$E(R) = \int_0^a r \frac{2r}{a^2} dr$	M1	Correct integral with limits (which may be implied subsequently).
		$= \left[\frac{2r^3}{3a^2}\right]_0^a$	A1	Correctly integrated.
		$=\frac{2a}{3}$	A1	Limits used. Accept unsimplified form.
		$E(R^2) = \int_0^a r^2 \frac{2r}{a^2} dr$	M1	Correct integral with limits (which may be implied subsequently).
		$= \left[\frac{2r^4}{4a^2}\right]_0^a = \frac{a^2}{2}$	A1	Correctly integrated and limits used. Accept unsimplified form.
		$a^{2} (2a)^{2} 9a^{2} - 8a^{2} a^{2}$	M1	Use of $Var(R) = E(R^2) - E(R)^2$
		$Var(R) = \frac{a^2}{2} - \left(\frac{2a}{3}\right)^2 = \frac{9a^2 - 8a^2}{18} = \frac{a^2}{18}$	A1	Convincingly shown; ANSWER GIVEN. Require sight of both terms expressed with a common denominator.
			[7]	
4	(iv)	$\overline{R} \sim \text{(approx) N} \left( \frac{2}{3} \times 22.5 = 15, \ \frac{22.5^2}{18 \times 100} = 0.28125 \right)$	B1	Normal.
		$\left(3^{22.5-15}, 18\times100^{-0.26125}\right)$	B1 B1	Mean. ft c's $E(R)$ (>0) with $a = 22.5$ . Variance. cao (= $0.5303(3)^2$ ) Accept unsimplified form.
			[3]	Variance. cao (= 0.3303(3)) Accept unsimplified form.
4	(v)	EITHER		
		can argue that 13.87 is more than 2 SD's from the Mean (15).	M1	Allow 1.96 SD's, but not 1.984.
		$15 - 2\sqrt{0.28125} = 13.93(9)$		1.96 gives 13.96(1). A 95% C.I. is (12.831, 14.909).
		<u>must</u> refer to $SD(\overline{R})$ , not $SD(R)$	M1	Must see explicit evidence for this.
		i.e. outlier		
		⇒ doubt.	A1	ft c's mean.
			[3]	
		<b>OR</b> more formally like a significance test: refer to $N(0,1)$	M1	
		$\frac{13.87 - 15}{\sqrt{0.28125}} = -2.131, \text{ sig at (eg) 5}\%$	M1	Could imply first M. $P( Z  > 2.131) = 0.0332$ .
		$\Rightarrow$ doubt.	A1	ft c's mean.

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# 4768 Statistics 3

### **General Comments**

There were 288 candidates (compared with 274 in January 2011) for this sitting of the paper. There were many very competent scripts and yet all too often candidates (including good ones) were seen to lose marks for carelessness, especially at the ends of questions. The topic "Sampling methods" continues to be one on which even good candidates do poorly. The numerical work was accurate except that candidates tended to quote confidence intervals to an excessive level of precision.

Invariably all four questions were attempted. Marks for Questions 1, 2 and 3 were found to be higher on average than Question 4.

### **Comments on Individual Questions**

- As mentioned above, candidates struggle on this topic. It was disappointing to see just how many did not appear to know the correct definition of a simple random sample. The usual (incorrect) answer given was "all members of the population are equally likely". There were also many who said "all samples are equally likely", omitting "of the required size". The most common "difficulty" suggested was that the sample would not be representative. This is more a consequence of taking a simple random sample rather than a difficulty with the process of obtaining one.
- **1 (b) (i)** Answers to this part of the question were usually fine. There were many completely correct solutions, and most marks lost were as a result of careless errors and/or omissions.
- Many completely correct answers were seen. Bizarrely there were candidates who, having scored full or nearly full marks for the test in the previous part, then switched away from the distribution  $t_{11}$  and so were penalised heavily. Many candidates insisted on quoting their final answer to a level of accuracy that was inappropriate.
- **2 (i)** This part, intended to be a straight-forward opener to the question, was answered correctly by the vast majority of candidates.
- Many candidates answered this correctly too, but at least as many interpreted the requirement "differ by 25" incorrectly, finding  $P(S_1 S_2 < 25)$  when they were meant to find  $P(-25 < S_1 S_2 < 25)$ . A noticeable few considered the sum of two small packets rather than the difference.
- Once again there were many correct answers to this part. It continues to be the case that candidates are not rigorous about the notation they use. For example, they insist on writing "2S" when they mean " $S_1 + S_2$ ", thus appearing not to know the difference, and in some cases it became obvious that they really did not.
- **2 (iv)** This was another part that was generally well-answered. Sometimes candidates got confused and addressed the wrong tail of the distribution.
- **2 (v)** This was mostly answered correctly. There were problems with excessive accuracy in the final answer and, occasionally, over the choice of the percentage point to use.

- For the chi-squared test, the hypotheses were broadly acceptable in most cases. Candidates usually realised to combine the last two classes in the table and so ended up with the correct value for the test statistic. However a very large proportion of them then failed to take account of the estimated parameter in the model and so used the wrong number of degrees of freedom to find the wrong critical value.
- **3 (b)(i)** A correct answer to this part was very rare indeed. A common answer went along the following lines, "There are differences between men and women and pairing eliminates these differences."
- **3 (b)(ii)** The Wilcoxon test was almost always done correctly but with just a bit more care needed over the wording of the final conclusion.
- 4 (i) Answers to this part were disappointing. Most candidates seemed unable to interpret symbolically the phrase "is proportional to". Many assumed, but without justification, that the required probability was the ratio of two areas.
- This was correct most of the time. Candidates who did not obtain the correct result had usually treated  $a^2$  as a variable, ending up with a complicated expression based on the quotient rule. Sometimes there was no indication of the variable with respect to which they would attempt to differentiate.
- **4 (iii)** For candidates who obtained the correct probability distribution function in part (ii), this part followed on quite easily, except that many were less than diligent about making sure that the final step towards the printed answer was shown carefully and convincingly. For the other candidates, if they knew what to do and carried it out carefully then most of the marks were still available.
- 4 (iv) Many correct answers were seen. However candidates often forgot to divide the variance by the sample size or they divided it by 10. The other flaw often seen was that candidates wrote " $R \sim ...$ " when it should have been " $\overline{R} \sim ...$ "
- **4 (v)** While a good number of well thought out answers were seen for this part, there were very many that lacked clarity and seemed not to have taken account of the preceding part.